Field linkage and magnetic helicity density

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Abstract

The helicity of a magnetic field is a fundamental property that is conserved in ideal MHD. It can be explored in the stellar context by mapping large-scale magnetic fields across stellar surfaces using Zeeman-Doppler imaging. A recent study of 51 stars in the mass range 0.1-1.34 M_{sun} showed that the photospheric magnetic helicity density follows a single power law when plotted against the toroidal field energy, but splits into two branches when plotted against the poloidal field energy. These two branches divide stars above and below ~0.5 M_{sun} We present here a novel method of visualising the helicity density in terms of the linkage of the toroidal and poloidal fields that are mapped across the stellar surface. This approach allows us to classify the field linkages that provide the helicity density for stars of different masses and rotation rates. We find that stars on the lower-mass branch tend to have toroidal fields that are non-axisymmetric and so link through regions of positive and negative poloidal field. A lower-mass star may have the same helicity density as a higher-mass star, despite having a stronger poloidal field. Lower-mass stars are therefore less efficient at generating large-scale helicity.

Helicity as a fundamental property

Magnetic helicity measures the amount of linkage and twist of field lines within a given volume. Since it is exactly conserved in ideal MHD and highly conserved for high magnetic Reynolds numbers in general (Woltjer 1958; Taylor 1974), helicity is an important factor when attempting to understand how magnetic fields are generated and evolve (e.g. Brandenburg & Subramanian 2005; Chatterjee et al. 2011; Pipin et al. 2019). Until recently, this could only be measured for the Sun (e.g. reviews by Démoulin 2007; Démoulin & Pariat 2009). We can, however, now map all three components of the large-scale magnetic field at the surfaces of stars using the spectropolarimetric technique of Zeeman-Doppler imaging (Semel 1989). While this does not allow us to determine the full helicity of a star's magnetic field (since we observe only at the stellar surface, not throughout the volume) it nonetheless provides valuable insights into the behaviour of stellar magnetic fields.

It may also hold the key to the puzzling lack of powerful mass ejections from stars – despite the abundance of powerful stellar flares. In the case of the Sun, coronal mass ejections are associated with the build-up of magnetic helicity. Is the relationship between magnetic energy and helicity perhaps different for other stars?

Mapping helicity densities across stellar surfaces

We represent the observed surface magnetic fields in terms of their poloidal and toroidal components. This avoids the need for a gauge (since the corresponding potential field with the same boundary flux is purely poloidal and therefore has zero helicity). Expansion of the magnetic field in terms of spherical harmonics gives the helicity density at the stellar surface in terms of the coefficients α_{lm} and γ_{lm} (the c_{lm} terms are normalisation factors):

$$h(\theta,\phi) = \mathrm{Re}\left(\sum_{lm}\sum_{l'm'} \frac{\alpha_{lm}\gamma_{l'm'}R_\star}{(l'+1)l(l+1)} c_{lm}c_{l'm'} e^{i\phi(m+m')} \left(P_{lm}P_{l'm'}\left(l(l+1) - \frac{mm'}{\sin^2\theta}\right) + \frac{\mathrm{d}P_{lm}}{\mathrm{d}\theta}\frac{\mathrm{d}P_{l'm'}}{\mathrm{d}\theta} \right) \right).$$

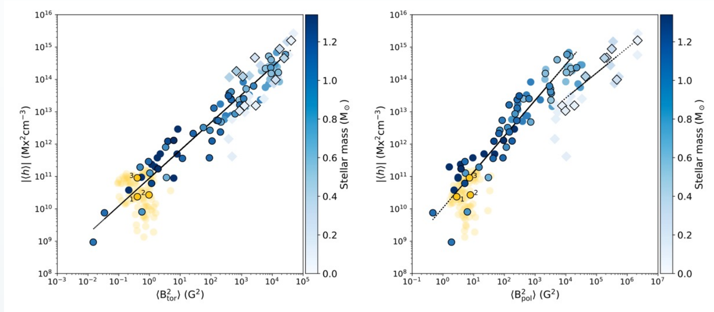

Fig. 1. Absolute helicity density for $l \leq 4$ averaged across a single hemisphere versus the mean squared toroidal (*left*) and poloidal (*right*) magnetic flux densities across the star. The colour of the symbols correspond to stellar mass, and the diamonds represent stars with $M_\star < 0.5\ M_\odot$. Symbols without an outline represent multiple measurements for the same stars, and the symbols with black edges are average values. The orange circles show the range of solar values, for the Southern hemisphere, between ~2010 and 2018.

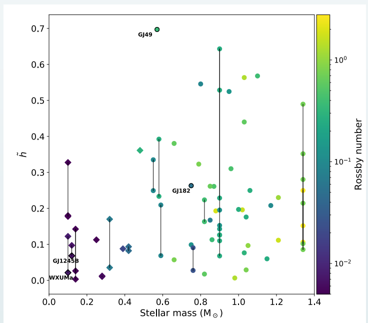

Fig. 2. The helicity energy fraction ($l \leq 4$), versus stellar mass. This describes the ratio of helicity to energy density. It is defined as $\tilde{h} \equiv |\langle h\rangle| / (R_\star \langle B^2\rangle)$.

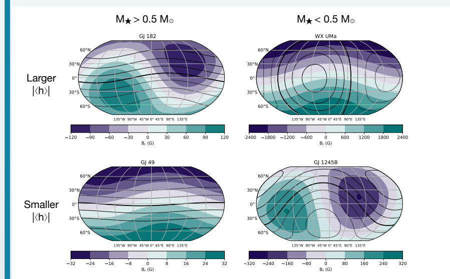

Fig. 3. A visualisation of the linkage of the dipole (l = 1) poloidal and toroidal field components of GJ 182, WX UMa (2008), GJ 49 and GJ 1245B (2006). The colour shows the strength of the radial magnetic (poloidal) field, and the black contours represent the toroidal magnetic field lines. The heavy black contour separates regions of positive (solid) and negative (dashed) toroidal field.

Distribution of stellar helicity densities

Using observations of 51 stars, Lund et al. (2020) calculated mean helicity densities in the observable hemisphere and mapped their distribution across the stellar surface.

They found that the helicity density scales with the toroidal energy according to $|\langle h\rangle| \propto \langle B_{tor}^2\rangle^{0.86 \pm 0.04}$ (Fig. 1, left). The scaling with the poloidal energy is more complex, however, revealing two groups with different behaviours (Fig 1, right). Stars less massive than ~ 0.5 $M_\odot$ are shown with diamond symbols. They appear to have an excess of poloidal energy when compared to more massive stars with similar helicity densities.

We can quantify this by calculating the ratio of the helicity density to the energy density (Fig. 2). For the lowest mass stars, this ratio is low, suggesting that they are inefficient producers of large-scale helicity (Lund et al 2021).

Why are low-mass stars poor helicity generators?

Helicity depends not simply on the magnetic field strength, but also the degree of linkage of the poloidal and toroidal fields. Even very strong stellar magnetic fields can have low helicity if there is little linkage of the poloidal and toroidal components.

We have developed a novel method to illustrate the degree of linkage of stellar magnetic fields (Fig. 3). The colour shows the strength of the radial (poloidal) magnetic field that passes *through* the stellar surface, while the black contours represent the toroidal field lines that lie *on* the stellar surface. If a loop of toroidal field encloses poloidal field of mixed polarity, the helicity produced is low. If, on the other hand, toroidal field loops enclose poloidal field of a single polarity, the helicity is larger.

In general, the maximum helicity density is achieved when the axes of symmetry of the poloidal and toroidal fields align. If they are offset by 90°, the helicity density is zero as there is no linkage between them.

Fig. 3 shows two pairs of stars: the stars in the top row each have similar, high helicity density while the stars in the bottom row each have similar, low helicity density. We show only the dipole (l=1) component for clarity. Note that the stars on the right (which belong to the low-mass branch in Fig. 1) have much stronger poloidal fields than their counterparts on the left (which belong to the higher-mass branch in Fig. 1).

Implications: Flares but few Coronal Mass Ejections?

Stars whose masses are low enough that they are likely to be fully-convective show a range of field strengths and hence a range of energies to power flares. However, they show correspondingly weak helicity densities.

The reason seems to be that their toroidal fields are typically asymmetric (relative to the rotation axis). As a result, they link through regions of both positive and negative poloidal field.

Low mass stars can therefore display powerful magnetic fields but low helicity density.

If the build-up of helicity drives mass ejections, this may explain the observed lack of powerful mass ejections from low mass stars that produce powerful flares.

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